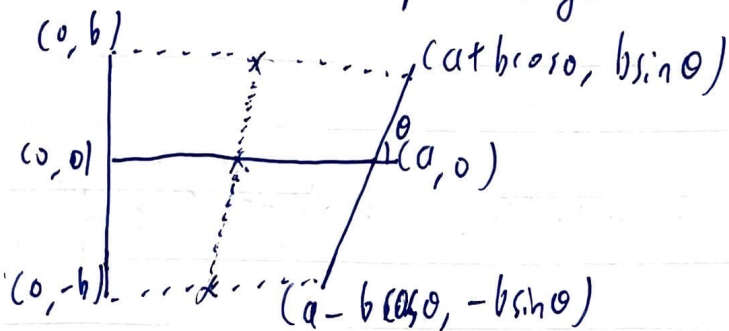


Prove

Q: For two arbitrary lines of same length, midpoints between them are concurrent.

without loss of generality, let



$$\text{midpoints} : \left(\frac{a - b \cos \theta}{2}, \frac{-b - b \sin \theta}{2} \right), \left(\frac{a}{2}, 0 \right), \left(\frac{a + b \cos \theta}{2}, \frac{b + b \sin \theta}{2} \right)$$

p_1 p_2 p_3

$$\text{line eq: } y - \alpha_i = m(x - \beta_i)$$

$$\cancel{y - y_{p_1}} \quad y - y_{p_1} = \frac{y_{p_3} - y_{p_1}}{x_{p_3} - x_{p_1}} (x - x_{p_1})$$

$$y + \frac{b + b \sin \theta}{2} = \frac{b + b \sin \theta}{b \cos \theta} \left(x - \frac{a - b \cos \theta}{2} \right)$$

$$\text{subs in } \left(\frac{a}{2}, 0 \right)$$

$$\begin{aligned} \text{L.H.S} &= \frac{b + b \sin \theta}{2} & \text{R.H.S} &= \frac{b + b \sin \theta}{b \cos \theta} \left(\frac{a}{2} - \frac{a - b \cos \theta}{2} \right) \\ & & &= \frac{b + b \sin \theta}{2} \end{aligned}$$

$$\therefore \text{L.H.S} = \text{R.H.S.} //$$